RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR
B.A./B.Sc. FOURTH SEMESTER (January – June) 2015
Mid-Semester Examination, March 2015

Date : 20/03/2015 **MATH FOR ECO (General)**
Time : 12 noon – 1 pm Paper : IV Full Marks : 25

[Use a separate answer book for each group]

Group – A

1. **Answer <u>any one</u> :** [1×5]

   a) Define Kernel of a linear transformation. Let $T:V \to W$ be a linear transformation. Prove that Ker T is a subspace of V. [1+4]

   b) A linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ is defined by
   $T(x_1,x_2,x_3)=(x_1+x_2+x_3, 2x_1+x_2+2x_3, x_1+2x_2+x_3), (x_1,x_2,x_3)\in\mathbb{R}^3$. Find Ker T and dimension of Ker T. [4+1]

2. **Answer <u>any two</u> :** [2×5]

   a) i) Prove that intersection of two subgroups of a group is a subgroup of the same group.
   ii) Prove that any group of prime order is cyclic. [2+3]

   b) i) State Lagrange's theorem.
   ii) Show that $\langle S,\bullet\rangle$ is an abelian group, where S is the set of permutations on the set {1,2,3,4} given by : $S=\left\{\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix},\begin{pmatrix}1&2&3&4\\2&1&3&4\end{pmatrix},\begin{pmatrix}1&2&3&4\\1&2&4&3\end{pmatrix},\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}\right\}$ & '•' is the composition of multiplication of permutations. [1+4]

   c) Show that A_3 is a normal subgroup of S_3. [5]

Group – B

Answer <u>any one</u> : [1×10]

3. a) For a 24-hour restaurant the following waitresses are required : [4]

Time of day	Minimum number of Waitresses
1 – 5	5
5 – 9	7
9 – 13	10
13 – 17	8
17 – 21	12
21 – 1	4

   Each waitress works eight consecutive hours per day. Find a linear programming model to find the smallest number of waitresses required to comply with the above requirements.

   b) Explain **<u>any three</u>** of the following terms : [3×2]
   i) linearly independent set
   ii) Spanning set
   iii) Basis
   iv) Basic feasible solution

4. a) Reduce the following linear programming problem in a standard maximization form :

Minimize $Z = 3x_1 - 4x_2 + 7x_3$

Subject to $x_1 + x_2 + 7x_3 \le 50$

$5x_1 + 3x_2 = 20$

$|3x_2 + 4x_3| \le 100$

where $x_1, x_2 \ge 0$ and x_3 is unrestricted in sign. [4]

b) Solve the following Linear Programming Problem by Simplex method.

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

and $x_1, x_2, x_3 \ge 0$ [6]

5. a) In the following L.P.P

Maximize $Z = 2x_1 - 4x_2 + 5x_3 - 6x_4$

Subject to $x_1 + 4x_2 - 2x_3 + 8x_4 + x_5 = 2$

$-x_1 + 2x_2 + 3x_3 + 4x_4 + x_6 = 1$

$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$

determine i) the maximum number of possible basic solutions

ii) the feasible extreme points

iii) the optimal basic feasible solution [1+2+2]

b) An intermediate table of an L.P.P by simplex method is given below in an incomplete form

			C_j		-2	0	0	0	
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
-4			$\frac{24}{5}$			$-\frac{2}{5}$	0	$\frac{1}{5}$	
$-M$		x_6	$\frac{18}{5}$			$\frac{1}{5}$	-1	$\frac{2}{5}$	
-2			$\frac{63}{5}$						
$Z_j - C_j$				0	0	$-\frac{M}{5}+\frac{6}{5}$	M	$-\frac{2}{5}M+\frac{2}{5}$	0

i) Complete the table

ii) Find the entering and departing vectors

iii) Write down the next table and show that the next table gives the unique optimal solution. [2+1+2]

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